WALL BOUNDARY LAYER IN A STRATIFIED MEDIUM

K. E. Dzhaugashtin* and A. Zh. Naimanova

UDC 532.558.21

An algebraic model of turbulence in a stratified medium is constructed using equations of second-order moments within the framework of a two-layer scheme.

In a turbulent flow of a stratified liquid in real situations, along with purely hydrodynamic forces Archimedean forces act. Here one of the basic problems is closing of the boundary-layer equations.

Prandtl established regularities in the distribution of various charactersitics of the flow as applied to a neutral-stratified medium. According to his "two-layer scheme" the entire boundary layer is subdivided into two sublayers, namely, a laminar sublayer adjacent to the underlying surface and a turbulent core of the flow. In the first sublayer, molecular transfer plays a major part, while in the second, turbulent transfer prevails. Further experimental investigations have revealed an additional intermediate layer between the sublayers mentioned in which a complicated nonlinear interaction of molecular and molar transfer modes exists. Subsequent theoretical studies have been aimed at creation of a semiemperical theory that provides a continuous velocity distribution within the framework of a unified one-layer model or a two-layer scheme that leads to a smooth change in the velocity from linear (in the laminar sublayer) to logarithmic (in the turbulent core of the flow).

Below, a semiempirical theory of the boundary layer in a stratified medium is presented that is based on balance equations of pulsation energy within the framework of a two-layer scheme and that contains a minimum number of virtually universal empirical constants. We note that this problem in such a formulation has not been considered previously in the literature.

We will investigate the boundary layer on a smooth infinite flat horizontal surface around which a uniform flow of an incompressible liquid passes under arbitrary vertical thermal stratification conditions.

The equations of motion and heat and the boundary conditions in the case under consideration are

$$\nu \frac{dU}{dz} + \langle - uw \rangle = u_*^2 , \quad a \frac{dT}{dz} + \langle - tw \rangle - \frac{q_w}{\rho c_p} ,$$
$$\nu \frac{dU}{dz} = \frac{\tau_w}{\rho} , \quad a \frac{dT}{dz} = \frac{q_w}{\rho c_p} \quad \text{at } z = 0 , \tag{1}$$

To close the system of equations (1), we use relations for one-point second-order moments and the velocity and temperature fields that take into account Archimedean forces [1]:

$$\frac{\partial}{\partial t} \langle u_i u_j \rangle + U_k \frac{\partial \langle u_i u_j \rangle}{\partial x_k} \langle u_j u_k \rangle \frac{\partial U_i}{\partial x_k} + \langle u_k u_i \rangle \frac{\partial U_j}{\partial x_k} =$$

$$= \frac{\partial}{\partial x_k} \left[\nu \frac{\partial}{\partial x_k} \langle u_j u_i \rangle - \langle u_i u_j u_k \rangle - \left\langle (\delta_{jk} u_i + \delta_{ik} u_j) \frac{P}{\rho} \right\rangle \right] +$$

* Deceased.

Institute of Theoretical and Applied Mathematics of the Science Ministry and of the Academy of Sciences of the Republic of Kazakhstan, Almaty. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 72, No. 2, pp. 294-300, March-April, 1999. Original article submitted November 11, 1997.

$$+\left\langle \frac{P}{\rho}\left(\frac{\partial u_j}{\partial x}+\frac{\partial u_i}{\partial x}\right)\right\rangle - 2\nu\left\langle \frac{\partial u_j}{\partial x}+\frac{\partial u_i}{\partial x}\right\rangle + \beta g\left(\delta_{3i}\langle tu_j\rangle + \delta_{3j}\langle tu_i\rangle\right),$$

$$\frac{\partial}{\partial t}\langle u_i t\rangle + U_k\frac{\partial\langle u_i t\rangle}{\partial x_k} + \langle u_k t\rangle\frac{\partial U_i}{\partial x_k} + \langle u_k u_i\rangle\frac{\partial T}{\partial x_k} =$$

$$= \frac{\partial}{\partial x_k}\left[\nu\frac{\partial}{\partial x_k}\langle u_i t\rangle - \langle u_i u_j t\rangle\left\langle\frac{P}{\rho}t\right\rangle\right] + \left\langle\frac{P}{\rho}\frac{\partial t}{\partial x_i}\right\rangle - 2\nu\left\langle\frac{\partial t}{\partial x_k}\frac{\partial u_i}{\partial x_k}\right\rangle - \beta g\delta_{3i}\langle t^2\rangle, \tag{2}$$

$$\frac{\partial}{\partial t}\frac{\langle t^2\rangle}{2} + U_k\frac{\partial\langle t^2\rangle}{\partial x_k} + \langle u_k t\rangle\frac{\partial T}{\partial x_k} = \frac{\partial}{\partial x_k}\left[\nu\frac{\partial}{\partial x_k}\langle t^2\rangle - u_k\frac{\langle t^2\rangle}{2}\right] - 2\nu\left\langle\frac{\partial t}{\partial x_k}\frac{\partial t}{\partial x_k}\right\rangle.$$

Equations (1) and (2) are written in generally accepted notation under the assumption of equality of the kinematic viscosity and the thermal diffusivity. To close (2), semiempirical relations of A. N. Kolmogorov and J. Rotta [2-4] are used:

$$2\nu\left\langle\frac{\partial u_j}{\partial x_k}\frac{\partial u_i}{\partial x_k}\right\rangle = c_1\nu\frac{\langle u_i u_j\rangle}{l^2} + \delta_{ij}\frac{2}{3}c\frac{E^{3/2}}{l},$$

$$2\nu\left\langle\frac{\partial t}{\partial x_k}\frac{\partial t}{\partial x_k}\right\rangle = c_{1t}\nu\frac{\langle t^2\rangle}{l^2} + c_t\frac{\langle t^2\rangle}{l}\sqrt{E},\quad 2\nu\left\langle\frac{\partial t}{\partial x_k}\frac{\partial u_i}{\partial x_k}\right\rangle = c_{ut}\nu\frac{\langle u_i t\rangle}{l^2}, \tag{3}$$

$$\left\langle\frac{P}{\rho}\left(\frac{\partial u_j}{\partial x_i}+\frac{\partial u_i}{\partial x_j}\right)\right\rangle = -k\frac{\sqrt{E}}{l}\left(\langle u_i u_j\rangle - \delta_{ij}\frac{2}{3}E\right),$$

$$\left\langle\frac{P}{\rho}\left(\frac{\partial t}{\partial x_i}\right)\right\rangle = -k_t\frac{\sqrt{E}}{l}\langle u_i t\rangle,\quad E = \frac{1}{2}\left(\langle u^2\rangle + \langle v^2\rangle + \langle w^2\rangle\right).$$

Here l is a quantity having the dimension of length; k, c (with various subscripts) are empirical constants determined below from experiments on turbulent flows in a homogeneous medium. It is assumed that the influence of stratification on their values is insignificant.

As experiments show, for turbulent flows of the type considered the diffusion terms in Eq. (2) can be neglected [4]. If we adopt this approximation, substitute (3) into (2), and write equations for purely shear flow ($u = U(z)$, $w = v = 0$, $T = T(z)$), we obtain a system of equations that in expanded form is

$$\langle uv\rangle\frac{dU}{dz} + \frac{k}{2}\frac{\sqrt{E}}{l}\left(\langle u^2\rangle - \frac{2}{3}E\right) + \frac{c_1}{2}\nu\frac{\langle u^2\rangle}{l^2} + \frac{c}{3}\frac{E^{3/2}}{l} = 0,$$

$$\frac{k}{2}\frac{\sqrt{E}}{l}\left(\langle w^2\rangle - \frac{2}{3}E\right) + \frac{c_1}{2}\nu\frac{\langle w^2\rangle}{l^2} + \frac{c}{3}\frac{E^{3/2}}{l} - \beta g\langle tw\rangle = 0,$$

$$\frac{k}{2}\frac{\sqrt{E}}{l}\left(\langle v^2\rangle - \frac{2}{3}E\right) + \frac{c_1}{2}\nu\frac{\langle v^2\rangle}{l^2} + \frac{c}{3}\frac{E^{3/2}}{l} = 0,$$

$$\langle w^2\rangle\frac{dU}{dz} + k\frac{\sqrt{E}}{l}\langle uw\rangle + c_1\nu\frac{\langle uw\rangle}{l^2} - \beta g\langle tu\rangle = 0,$$

$$\langle vw\rangle \frac{dU}{dz} + k\frac{\sqrt{E}}{l}\langle uv\rangle + c_1\nu\frac{\langle uv\rangle}{l^2} = 0\,,$$

$$k\frac{\sqrt{E}}{l}\langle vw\rangle + c_1\nu\frac{\langle vw\rangle}{l^2} - \beta g\langle tv\rangle = 0\,, \tag{4}$$

$$\langle tw\rangle \frac{dU}{dz} + \langle uw\rangle\frac{dT}{dz} + k_t\frac{\sqrt{E}}{l}\langle tu\rangle + c_{ut}\nu\frac{\langle tu\rangle}{l^2} = 0\,,$$

$$\langle w^2\rangle \frac{dT}{dz} + k_t\frac{\sqrt{E}}{l}\langle tw\rangle + c_{ut}\nu\frac{\langle tw\rangle}{l^2} - \beta g\langle t^2\rangle = 0\,,$$

$$\langle wv\rangle \frac{dT}{dz} + k_t\frac{\sqrt{E}}{l}\langle tv\rangle + c_{ut}\nu\frac{\langle tv\rangle}{l^2} = 0\,,$$

$$\langle tw\rangle \frac{dT}{dz} + c_{1t}\nu\frac{\langle t^2\rangle}{l^2} + c_t\frac{\langle t^2\rangle\sqrt{E}}{l} = 0\,.$$

Thus, we have a system of algebraic equations relative to one-point second-order moments ($\langle u_iu_j\rangle$, $\langle tu_i\rangle$, $\langle t^2\rangle$).

Note that the equations contain quantities proportional to the physical viscosity ν that are absent in all other models of turbulent flow in a stratified medium. Precisely the presence of these terms makes it possible to include in calculation a buffer zone between the laminar sublayer and the turbulent core of the flow, as will be done below, within the framework of the two-layer scheme.

We now evaluate the empirical constants [5-7]. If we solve the equations for a developed turbulent flow in a homogeneous medium [5, 6], we will obtain the following expressions for turbulent transfer of momentum and heat and the anisotropy coefficient:

$$-\langle uw\rangle = \left(\frac{2}{3}\left(\frac{k}{c}-1\right)^{1/2}\right)\frac{c}{k^3}\left(l\frac{dU}{dz}\right)^2,\quad \langle tw\rangle = \frac{k}{k_t}l^2\frac{dU}{dz}\frac{dT}{dz}\,,$$

$$\frac{\langle u^2\rangle}{\langle w^2\rangle} = \frac{k/c+2}{k/c-1}\,. \tag{5}$$

Since l is determined by the adopted hypotheses with an accuracy up to a constant multiplier, the numerical coefficient in the expression for $-\langle uw\rangle$ can be assumed to be equal to unity:

$$\left(\frac{2}{3}\left(\frac{k}{c}-1\right)^{1/2}\right)\frac{c}{k^3} = 1\,.$$

Hence it follows that k and c are determined by the single constant k/c, whose value is found from experimental data on the finite degeneracy of the turbulence behind a grid [4]. They show that $\langle u^2\rangle/\langle w^2\rangle \simeq 1.5$. Next from (5) it follows that

$$k/c \simeq 7\,. \tag{6}$$

TABLE 1. Pulsation Characteristics in Parametric Relation to R_L, R_E, Gr

F	F_i	Φ_i	i
$\langle u^2\rangle$	E	$2\dfrac{mnp(m-\lambda)+(m^2-\lambda m+2mp-p\lambda)\mathrm{Gr}}{m(mnp+(m+2p)\mathrm{Gr})}$	1
$\langle w^2\rangle$	E	$\dfrac{\lambda(np+\mathrm{Gr})}{mnp+(m+2p)\mathrm{Gr}}$	2
$\langle v^2\rangle$	E	$\dfrac{\lambda}{m}$	3
$-\langle uw\rangle$	$\nu\dfrac{dU}{dz}$	$\dfrac{\lambda(n^2p+(n-p)\mathrm{Gr})\mathrm{R}_E^2}{(mn+\mathrm{Gr})(mnp+(m+2p)\mathrm{Gr})}$	4
$-\langle tw\rangle$	$a\dfrac{dT}{dz}$	$\dfrac{\lambda p\mathrm{R}_E^2}{mnp+(m+2p)\mathrm{Gr}}$	5
$\langle t^2\rangle$	$\left(l\dfrac{dT}{dz}\right)^2$	$\dfrac{\lambda\mathrm{R}_E^2}{mnp+(m+2p)\mathrm{Gr}}$	6
$\langle tu\rangle$	$l^2\dfrac{dU}{dz}\dfrac{dT}{dz}$	$\dfrac{\lambda pn(m+n)+\lambda(p+n^2-pn)\mathrm{Gr}}{n(mn+\mathrm{Gr})(mnp+(m+2p)\mathrm{Gr})}$	7

Then k and c are

$$k=\left(\frac{c}{k}\right)^{1/2}\left(\frac{2}{3}\left(\frac{k}{c}-1\right)^{1/2}\right)\simeq 1.7\,;\quad c=\left(\frac{c}{k}\right)^{3/2}\left(\frac{2}{3}\left(\frac{k}{c}-1\right)^{3/4}\right)\simeq 0.15\,.$$

The ratio k/c in (6) satisfactorily describes experiments on a flow of a homogeneous medium in a channel [6] and a flow in a wake behind a body and stratified jet flows [7]. The expression for the turbulent Prandtl number $\sigma_t = k/k_t$ determined in [8] from experimental data in a homogeneous medium gives $\sigma_t \simeq 0.75$.

The other coefficients are determined from the theory of degeneracy of the dynamic and scalar fields of isotropic turbulence [4] and are approximately

$$c_t/c\simeq 1\,,\quad c_1=\frac{5}{4}\pi\,,\quad c_{1t}=\frac{3}{4}\pi\sigma\,,\quad c_{ut}=\pi\sigma\,,\quad \sigma=\frac{\nu\kappa}{a}\,.$$

The constants c_1 and c_{ut} will next be corrected in accordance with experimental data on the velocity and temperature distribution in a homogeneous medium since in real flows on a flat surface the turbulence structure has an anisotropic nature.

For convenience in solving Eqs. (4) we introduce the following notation:

$$m=k\mathrm{R_E}+c_1\,,\quad q=c\mathrm{R_E}+c_1\,,\quad \lambda=\frac{2}{3}(k-c)\,\mathrm{R_E}\,,\quad p=c_t\mathrm{R_E}+c_{1t}\,,\quad n=k_t\,\mathrm{R_E}+c_{ut}\,,$$

$$\mathrm{R_L}=l^2\frac{dU}{dz}\,,\quad \mathrm{R_E}=l\frac{\sqrt{E}}{\nu}\,,\quad \mathrm{Ri}=\beta g l^2\frac{dT}{dz}\Big/\left(\frac{dU}{dz}\right)^2,\quad \mathrm{Gr}=\mathrm{Ri}\,\mathrm{R_L^2}\,, \tag{7}$$

where R_L is the local Reynolds number characterizing energy generation; R_E is the turbulent Reynolds number determined by the value of the perturbation energy.

Now we rewrite (4) with account for notation (7):

$$\mathrm{R_L}\frac{\langle uw\rangle}{E}+q-\beta g\frac{l^2}{\nu}\frac{\langle tw\rangle}{E}=0\,,\quad m\frac{\langle w^2\rangle}{E}-\lambda-2\beta g\frac{l^2}{\nu}\frac{\langle tu\rangle}{E}=0\,,$$

$$m\frac{\langle w^2\rangle}{E}-\lambda=0\,,\quad m\langle uw\rangle+\mathrm{R_L}\langle w^2\rangle-\beta g\frac{l^2}{\nu}\langle tu\rangle=0\,,$$

$$n\langle tu\rangle+\mathrm{R_L}\frac{\langle tw\rangle}{E}\frac{l^2}{\nu}\frac{\partial T}{\partial z}\langle uw\rangle=0\,,\quad n\langle tw\rangle+\frac{l^2}{\nu}\frac{\partial T}{\partial z}\langle w^2\rangle-\beta g\frac{l^2}{\nu}\langle t^2\rangle=0\,,\tag{8}$$

$$p\langle t^2\rangle+\frac{l^2}{\nu}\frac{\partial T}{\partial z}\langle tw\rangle=0\,,\quad \langle v^2\rangle=\langle w^2\rangle\,,\quad \langle uw\rangle=\langle vw\rangle=0\,,$$

$$E=(\langle u^2\rangle+\langle v^2\rangle+\langle w^2\rangle)/2\,.$$

The solution of (8) is given in Table 1, where the sought expressions for the one-point second-order moments are presented in the form of two cofactors $F = F_i\Phi_i$, $i=\overline{1,7}$, where Φ_i takes account of the influence of stratification of the medium.

Thus, the obtained expressions for the one-point second-order moments make it possible to close the system of equations (1) and calculate, correspondingly, the velocity and temperature fields.

Passing, as usual, to the universal variables

$$\eta=\frac{zu_*}{\nu}\,,\quad \varphi=\frac{U}{u_*}\,,\quad \theta=\frac{T}{T_*}\,,\quad T_*=-\frac{1}{\kappa u_*}\frac{q_w}{\rho c_p}\,,\tag{9}$$

we write (1) in the form

$$\frac{d\varphi}{d\eta}=\frac{1}{1+\Phi_4(\mathrm{R}_E)}\,,\quad \frac{d\theta}{d\eta}=\frac{\sigma}{1+\Phi_5(\mathrm{R}_E)}\,.\tag{10}$$

Within the framework of the two-layer scheme, in the laminar sublayer Archimedean forces do not exert an influence on the regularities of the velocity and temperature distributions. Therefore the equations of motion and heat can be written as

$$\frac{d\varphi}{d\eta}=1\,,\quad \frac{d\theta}{d\eta}=\sigma\,,$$

On a smooth plate at $\eta = 0$ the velocity and the temperature are equal to zero: $\varphi = 0$, $\theta = 0$. Thus, the distributions of the sought quantities are linear [9]: $\varphi = \eta$, $\theta = \sigma\eta$.

The stratification of the medium influences the thickness of the laminar sublayer. To determine it, we consider a graphical analog of the energy balance equation (which is obtained by summation of the first three equations of system (8)):

$$\varphi_{\mathrm{I}}=\varphi_{\mathrm{II}}\,,\quad \varphi_{\mathrm{I}}=(A_4+A_5\,\mathrm{Gr})\,\mathrm{R_L^2}\,;\quad \varphi_{\mathrm{II}}=A_1+A_2\,\mathrm{Gr}+A_3\,\mathrm{Gr}^2\,,\tag{11}$$

where $A_1 = pq(mn)^2$; $A_2 = mn(\lambda p + qm + 3pq)$; $A_3 = \lambda p + qm + 2pq$; $A_4 = \lambda n^2 p$; $A_5 = \lambda(n - p)$.

Equation (11) is the energy balance of disturbing motion, the left-hand side of which is proportional to energy generation, and the right-hand side, to dissipation. According to the analysis made in [2], at some value $\mathrm{R_L} = \mathrm{R_{L0}}$ the flow mode changes. At this point relation (11) and the additional relation [2]

$$\frac{d\varphi_{\mathrm{I}}}{d\mathrm{R_E}}=\frac{d\varphi_{\mathrm{II}}}{d\mathrm{R_E}}\,,\tag{12}$$

must be fulfilled, i.e., $(A_4' + A_5'\mathrm{Gr})\mathrm{R_L^2} = A_1' + A_2'\mathrm{Gr} + A_3'\mathrm{Gr}^2$, where a prime denotes differentiation of the function A_i with respect to $\mathrm{R_E}$.

Thus, Eqs. (11), (12) serve to determine the critical values of the local numbers $\mathrm{R_L^*}$ and $\mathrm{R_E^*}$.

To integrate system (10), it is necessary to find a relationship between the variables η and R_E. For this, we turn to the expression for the local number R_L. Taking in it the Prandtl mixing path length $l = \kappa\eta$ as a scale and passing to the universal variables, we obtain

$$R_L = (\kappa\eta)^2 \frac{d\varphi}{d\eta}.$$

Replacing R_L here by the expression for it obtained from the energy balance equation and substituting for the derivative $d\varphi/d\eta$, we obtain from (10)

$$\eta = \frac{1}{\kappa}\sqrt{R_L(1 + \Phi_4(R_E))}. \tag{13}$$

In the transition region near the wall the viscosity is among the governing parameters, and therefore from dimensionality considerations it follows that $lu_*/\nu = f(\eta)$. Next, the linear dependence $lu_*/\nu = \kappa_1\eta$ is assumed. From the matching conditions at the interface of transition region–turbulent core we have $\kappa_1 = \kappa$ [6].

The right-hand side of expression (12) is a function of R_E, i.e., $\eta = \eta(R_E)$. Therefore we pass to the new variable R_E in the equations of motion and heat. As mentioned above, values of the local Reynolds numbers that exceed certain critical quantities (denoted by an asterisk) correspond to the turbulent mode of flow:

$$R_E > R_E^*,\quad R_L > R_L^*,\quad Ri > Ri^*.$$

Lower values of R_E, R_L, and Ri correspond to laminar flow. In this case we must discard the turbulent shear stress of friction ($-\langle uw\rangle = 0$) and heat transfer ($-\langle tw\rangle = 0$) in system (1). Then this system can be written in the universal variables in a form corresponding to the laminar sublayer.

The boundary of the viscous sublayer η^* and the velocity and temperature at this boundary are determined from the critical values R_E^*:

$$\eta^* = \frac{1}{\kappa}\sqrt{R_L^*(1 + \Phi_4(R_E^*))},\quad \varphi^* = \eta^*,\quad \theta^* = \sigma\eta^*.$$

In the new variables Eq. (11) is written as

$$\frac{d\varphi}{dR_E} = \frac{d\eta}{dR_E}\frac{1}{1 + \Phi_4(R_E)},\quad \frac{d\theta}{dR_E} = \frac{d\eta}{dR_E}\frac{1}{1 + \Phi_5(R_E)}.$$

Integration of it with the initial conditions

$$R_E = R_E^*,\quad \varphi = \varphi^*,\quad \theta = \theta^*$$

yields

$$\varphi = \int_{R_E^*}^{R_E} \frac{d\eta}{dR_E} = \frac{1}{1 + \Phi_4(R_E)}\, dR_E,\quad \theta = \int_{R_E^*}^{R_E} \frac{d\eta}{dR_E}\frac{\sigma}{1 + \Phi_5(R_E)}\, dR_E;$$

$$\eta = \frac{1}{\kappa}\left(\frac{A_1 + A_2\,Gr + A_3\,Gr^2}{A_4 + A_5}\right)^{1/4}(1 + \Phi_4)^{1/2}, \tag{14}$$

where $Gr = Gr_0(\kappa\eta)^4\sigma/(1 + \Phi_5)$; $Gr_0 = \beta g\nu T_*/u_*^3$.

Expressions (14) determine the velocity and temperature profiles in quadratures in parametric relation to R_E.

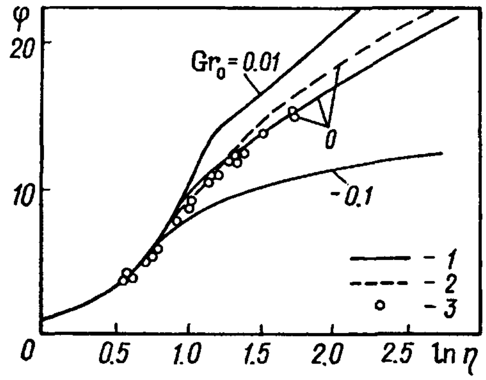
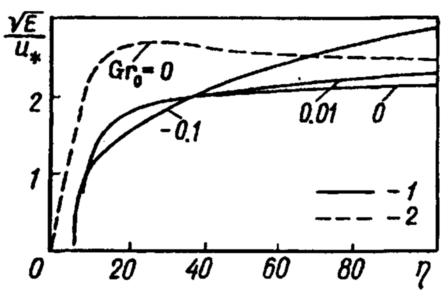


Fig. 1. Universal velocity profile for different values of the stratification parameter Gr_0: 1) calculation (c_l = 2.3), 2) calculation (c_l = 3.92), 3) experimental data [10].



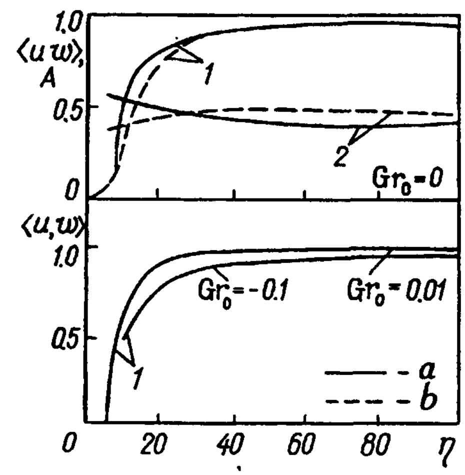


Fig. 2. Distribution of the turbulent pulsations $\sqrt{E}/u_*$ in the transition region near the wall for different values of the stratification parameter: 1) calculation, 2) data of [10].

Fig. 3. Profiles of the turbulent friction (curves 1) and the correlation coefficient $\langle uw\rangle/(\langle u^2\rangle\langle w^2\rangle)^{1/2} \equiv A$ (curves 2) for different values of the stratification parameter Gr_0: a) calculation, b) data of [10].

In solving Eqs. (14), the boundary of the viscous sublayer was preliminarily determined from (13) using the critical values R_L^*, R_E^*, which, in turn, were found from (11) and (12). Then the integrals in (14) determining the universal velocity and temperature profiles were calculated.

The results calculated are given in Figs. 1-3.

As noted above, the values of the constants c_l, c_{ut} require correction. For this purpose we will consider the universal velocity profile in a homogeneous medium (Gr_0 = 0). As seen from Fig. 1, the dashed curve with c_l = 3.92 is above the calculated (solid) curve and the experimental data of Layfer [10]. The choice of c_l = 2.25 provides coincidence of the velocity profiles. The value of c_l is close to the quantity obtained by Levin (c_l = 2.4). For correction of c_{ut} we assume σ_t = 1. In this case the profile of θ for Gr_0 = 0 must coincide with φ (c_l = 2.25). The best correspondence of them is provided by the choice of c_{ut} = 2.617.

Figure 1 also presents velocity profiles for different values of the stratification parameter Gr_0 (solid curves).

In Figs. 2, 3 the turbulent friction, the correlation coefficient $\langle uw\rangle/(\langle u^2\rangle\ \langle w^2\rangle)^{1/2}$, and the turbulent pulsations $\sqrt{E}/u_*$ in the transition region near the wall are compared to the experimental Layfer data of [10] for neutral stratification. Here, curves calculated for stratification conditions are also given. Stable stratification suppresses the turbulent friction, while unstable stratification, on the other hand, increases it. Under unstable stratification conditions the turbulent pulsations $\sqrt{E}/u_*$ are increased, but in the case of stable stratification their increase is insignificant.

NOTATION

Ri, Richardson number; Gr, local Grashof number; δ_{ij}, Kronecker symbol; ν, kinematic viscosity; ρ, density; P, pressure fluctuation; κ, Kármán constant; τ_w, friction stress on the wall; q_w, heat flux on the wall; c_p, heat capacity at constant pressure; σ_t, turbulent Prandtl number; l, mixing path length; g, gravitational acceleration; t, temperature fluctuation; (u, v, w), components of the vector of the pulsation velocity; U, mean longitudinal component of the velocity; T, mean temperature; β, coefficient of thermal expansion; E, kinetic energy of the pulsation; $u_* = (\tau_w/\rho)^{1/2}$, dynamic velocity; a, thermal diffusivity.

REFERENCES

1. A. S. Monin and A. M. Yaglom, Statistical Hydromechanics [in Russian], Pt. 1, Moscow (1965).
2. K. E. Dzhaugashtin, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 156-159 (1979).
3. J. Rotta, Z. Physik, 129, No. 6, 547-572 (1951).
4. J. O. Hintse, in: Turbulence. Its Mechanism and Theory (ed. G. A. Abramovich) [Russian translation], Moscow (1963).
5. B. A. Kolovandin, in: Heat and Mass Transfer (ed. A. V. Luikov) [in Russian], Vol. 1, Moscow (1968), pp. 154-167.
6. V. B. Levin, Teplofiz. Vys. Temp., 2, No. 4, 588-598 (1964).
7. K. E. Dzhaugashtin and A. A. Shelepov, Izv. Sib. Otd. Akad. Nauk SSSR, Fiz.-Tekh. Zh., Issue 6, 42-45 (1991).
8. Z. B. Sakipov, Izv. Akad. Nauk Kaz. SSR, Ser. Énerg., Issue 1(19) (1961).
9. L. G. Loitsyanskii, Mechanics of Liquid and Gas [in Russian], Moscow (1987).
10. J. Layfer, "The structure of turbulence in fully developed pipe flow," Nat. Advis. Com. Aeronaut. Rep. No. 1174 (1954).